# Examining the role of perception, social and private information in honey bee foraging algorithms

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Abstract: The last decade has seen the development of a family of powerful optimisation algorithms inspired by the foraging behaviours of honey bees. A key component of these algorithms is the concept of 'recruitment' whereby successful foragers transmit information to other colony members concerning the location of discovered resources and thereby 'recruit' for that location. However, real-world foraging by honey bees is much more complex and embeds a number of additional features, including individual sensory 'perception', 'noisy' recruitment, and 'private information' (memory). In this study, we develop a series of algorithms which embed these features and assess the impact of each on the effectiveness of the resulting search performance on a series of benchmark problems representing differing resource landscapes. The simulation results support findings from the empirical study of real-world honey bees that recruitment is not a unique driver of successful foraging activity and that private information also plays an important role. This finding is relevant for the design of honey bee optimisation algorithms.

**Keywords:** honey bee algorithms; social foraging; optimisation; social information; private information; perception.

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#### 1 Introduction

A substantial literature has developed in the area of swarm intelligence since the 1990s (Bonabeau et al., 1999; Dorigo, 1992). This literature seeks to gain understanding about the mechanisms by which order and problem solving behaviours can emerge in societies of agents, when decision-making is decentralised and the result of a bottom-up rather than a top-down process. An example of this is provided by the activities of many species of ants and bees which exhibit remarkable ability to coordinate their activities in a bottom-up manner. One particular behaviour in both families of insects that has attracted much study is that of food-foraging.

Foraging is obviously an important activity for all organisms and its organisation can range from solitary foraging, where an individual forages on its own, to social foraging where foraging is a group behaviour. The essence of social foraging is that there must be some communication between organisms. Communication about resource finds may take place between individuals, via environment marking, or via a broadcast mechanism at a communal nest or hive (Sumpter and Brannstrom, 2008).

#### The observation that foraging requires organisms to undertake a search process has in turn led to the design of several families of search algorithms which draw metaphorical inspiration from a range of real-world foraging behaviours. These include ant colony optimisation algorithms (Bonabeau et al., 1999; Dorigo, 1992; Dorigo and DiCaro, 1999; Dorigo et al., 1996; Dorigo and Stützle, 2004) and honey bee algorithms (Bansal et al., 2013; Chong et al., 2006; Nakrani and Tovey, 2004; Karaboga, 2005; Pham et al., 2006; Yang, 2005).

In the case of both ant and honey bee foraging, several species of the insects are *central place foragers* in that they return to a colony or hive in order to store food. Therefore, they can interact with colony members and potentially pass on information about food finds. A notable aspect of their interaction is that successful foragers seek to *recruit* other conspecifics to food resources that they have found. This social transmission of information is emphasised in the majority of search algorithms inspired by foraging processes. However, real-world foraging processes are multi-layered and embed a variety of mechanisms including perception, memory and errorful learning from conspecifics.

In this paper, we describe some features of real-world honey bee foraging which have not attracted explicit attention in previous work on the design of honey bee algorithms and seek to determine the impact of each of these features on search performance using a series of benchmark optimisation problems.

The remainder of this contribution is organised as follows. Section 2 provides some background on honey bee behaviours and provides a synopsis of some recent developments in our understanding of these behaviours. Section 3 describes the experiments undertaken in this study. The results are presented and discussed in Section 4 and finally, conclusions and opportunities for future work are discussed in Section 5.

#### 2 Background

European honey bees (*Apis mellifera*) are one of the most-studied branches of the insect family. Just as in the case of certain species of ants, their ability to self-organise in complex ways has long attracted the attention of researchers who have examined the mechanisms of communication in honey bee societies. Apis mellifera exhibit a symbolic system of communication based on the performance of a *dance* to transmit information on (amongst other things) the location and quality of resources in the vicinity of their hive. The dance language of these honey bees is remarkably complex and was first decoded by Karl von Frisch who was subsequently awarded the Nobel Prize for physiology and medicine in 1973 for this work.

#### 2.1 The honey bee recruitment dance

Foraging activities of bees involve searching for exploitable resources such as pollen (a source of protein), water, waxy materials for hive building, and nectar from flowers. Nectar, a source of carbohydrate, is converted by bees into honey. When a scout or explorer bee discovers a food source of sufficient quality she may undertake a dance on its return to the hive once she has unloaded her nectar. The objective of the dance is to recruit other foragers who will travel to the food source and exploit it. In turn, the newly-recruited bees may also undertake a dance when they return to the hive if the food resource is of sufficient quality.

The dance language consists of repetitive patterned movements that are intended to communicate information about the location and desirability of the food source. In essence, the dance can be considered as a re-enactment of the flight from the hive to the location of interest. The dance is undertaken in a specific location of the hive near the entrance called the *dance floor*. The dance floor consists of a vertical comb in the hive and typically this area of the hive contains multiple potential foraging recruits. The dance is social in that it is never undertaken without an audience (Crist, 2004).

The nature of the dance movements depends on the location of the food source relative to the hive. If the food source is close-by (up to about 100 metres from the hive), the bees undertake *round dances* with circular movements predominating. If the food source is further away a *waggle dance* resembling a figure eight is undertaken. The direction to the resource (relative to the sun) is indicated by the angling of the bee's body during the dance. The desirability of the location is communicated by the dance's *liveliness* or *enthusiasm*, with more desirable locations corresponding to livelier dances (Seeley et al., 2000). The duration of the waggle portion of the dance is a proxy for the distance to the location of further away food sources.

At any point in time there may be several bees dancing on the dance floor, hence, the hive can simultaneously harvest several food sources. This permits quick adaptation by the bee colony in the event that a particular food resource becomes exhausted and therefore needs to be abandoned. Recruited foragers tend to travel to richer food sources in greater numbers as dances for high-quality food sources tend to be more conspicuous and longer, thereby creating a positive feedback loop resulting in the amplification of the exploitation of those food sources. The above description of foraging behaviour is stylised and omits aspects of the recruitment process such as the role of odours and sounds. Readers requiring detail on these aspects are referred to Seeley (1995), Tereshko and Lee (2002), and von Frisch (1967).

It is noted that like ant-colonies, decision making by bee colonies, is decentralised and parallel. However, there are notable distinctions between the information-sharing mechanisms of ants and those of bees. Communication between ants is primarily indirect and is based on stigmergy. In contrast, the honey bee dance language enables bees to engage in direct communication whereby information is symbolically broadcast to peers by means of a dance.

#### 2.2 Bee nest site selection

Another example of cooperative problem solving via recruitment by honey bees is provided by nest site selection. Typically, in late spring or early summer as a colony outgrows its current hive, the colony will *fission* or 'divide' whereby the queen bee and about half of the population of worker bees will leave the hive and seek to establish a colony at a new nest-site, leaving behind a young queen bee and the remainder of the worker bees in the existing hive. Having left the current hive, the swarm usually do not fly far and within about 20 minutes they form a football-sized cluster of bees, often on the branch of a tree (Beekman et al., 2006). From this location, *scout* bees begin a search for a new nest site.

During the site selection process, scout bees leave the cluster and search for a new nest site. As potential nest sites of satisfactory quality are uncovered, the returning scout bees communicate their location to other scout bees by doing a waggle dance on the surface of the swarm. The length of the dance depends on the quality of the site found, with longer dances being undertaken for better-quality sites. If a bee finds a good site, it becomes *committed* to it and will visit it several times. However, the length of its recruitment dance for the site will decrease after each visit. This phenomenon is known as *dance attrition* (Seeley et al., 2006).

The net effect of the recruitment and the dance attrition phenomena is that higher-quality sites attract more attention from the searching scouts creating a positive reinforcement cycle. Dance attrition facilitates the 'forgetting' of nest-site locations that are not continually reinforced, akin to pheromone evaporation in ant-colony foraging. While multiple nest-sites (if several of sufficient quality exist) will be considered in the early stage of the search process, these will be quickly whittled down to a limited number of choices from which one is finally chosen. Unlike the foraging process whereby several food locations may be harvested simultaneously, the nest-site selection problem produces a single 'winner'.

#### 2.3 Developments in the honey bee literature

Honey bee foraging behaviours are a rich source of inspiration for the design of computational algorithms, and are a rapidly growing sub-field of natural computing. In most foraging-inspired honey bee algorithms, the core concept is that of recruitment, whereby bees which have found good food sources recruit conspecifics which travel to, and harvest, the food resource (or 'good' location on the fitness landscape). In order to avoid premature convergence and maintain diversity in the search process, 'forgetting' mechanisms are typically included in algorithmic implementations. These can be as simple as the maintenance of continual random search by some foragers. Whilst the resulting algorithms have proven to be highly effective for optimisation, they incorporate a limited number of features of the full behavioural repertoire of honey bees. These behaviours have been extensively studied in recent decades and we now possess a much more comprehensive understanding of the foraging process of honey bees. Drawing on this literature, three items in particular are noteworthy:

- 1 individual perception
- 2 noisy-recruitment
- 3 the role of private information.

#### 2.3.1 Individual perception

Honey bees have visual sensory capabilities (Srinivasan et al., 1996; Srinivasan, 2010) and were the second nonhuman organism (after fish) for which colour vision was demonstrated (Srinivasan, 2010). Their visual abilities extend beyond recognition of simple patterns, colours or shapes, and recent work has shown that their visual acuity is even sufficient to distinguish between Monet (Impressionist)/Picasso (Cubist) paintings (Wu et al., 2013), and between human faces (Dyer et al., 2005).

Specific features of honey bee vision are described by Morawetz and Spaethe (2012). In contrast to bumblebees which process information in parallel from a wide visual field, honey bees process visual information in a serial-like search behaviour, with search terminating the moment the first target is uncovered. Honey bees have a smaller visual field than bumble bees and hence, their visual search is more akin to "moving a small spotlight step by step over the search area" [Morawetz and Spaethe, (2012), p.2522]. Given these abilities, honey bees are capable of identifying promising food sources at a distance and altering their flight trajectory to forage at this resource. Individual perception has not been explicitly included in honey bee optimisation algorithms to date.

#### 2.3.2 Noisy-recruitment

A second issue is that the recruitment dances of honey bees is much 'noisier' in the real-world than is typically suggested in honey-bee algorithms. A dancing bee will repeat the dance multiple times, sometimes up to 100 times (Toufailia et al., 2013), with higher quality sources tending to induce more dancing behaviour. Observational evidence indicates that repeated dances for a specific resource by the same, or by different bees recruiting for the same resource, often vary in both directional and distance information (Gruter et al., 2013; Toufailia et al., 2013). Hence, dances only recruit to an approximation of the location of the food resource. Dance followers observe several iterations of the dance to compute an average vector with some 5-10 performances being observed at a minimum. Despite this, most recruits have to undertake several trips before finding the advertised food source. On average in only some 12% to

25% of cases does dance following lead to the discovery of the advertised resource (Biesmeijer and Seeley, 2005).

Counter-intuitively, observational evidence indicates that noise concerning the directional component of the vector is larger for nearby food sources (within 1 km from the nest) than for more distant food locations. The distance component of the vector is also noisy with the variance of the distance information being scale-invariant (De Marco et al., 2008) again indicating greater noise for close-by food sources. One possible explanation for this phenomenon is that given the typical spatial configuration of bees' food sources (flower clumps in sizeable patches), noisy directional information serves to spread recruits over an area of food resources rather than recruiting all bees to the exact same foraging location which is being harvested by the dancing bee.

A side effect of a noisy recruitment signal is that foragers who have been newly recruited to a foraging location will not be sure of the exact energy requirements of the foraging flight. In the normal course of events, honey bee foragers take small amounts of honey from nestmates via trophallaxis before leaving the hive in order to provide energy (fuel) for their flight. A study by Harano et al. (2013) found that dance followers carried a larger amount of honey than dancers – but this differential reduced over repeated trips to the same food location. This represents a physical manifestation of the location uncertainty faced by newly-recruited foragers.

Although, prima facie, a noisy communication mechanism would appear sub-optimal, it has been suggested that the imprecision in the honey bee dance could in fact be adaptive as it would allow for the discovery and exploitation of food sources which are nearby to the resource originally recruited for (Granovskiy et al., 2012). In essence, it injects a stochastic element into the foraging process.

#### 2.3.3 Private information

A third issue is that in spite of the importance accorded to recruitment in most honey bee algorithms, real-world bees place substantial reliance on personal (private) information, instead of socially-acquired information from observing a dance, when engaging in foraging. Over the foraging lifetime of a honey bee (approximately 99.5 +/– 27.3 foraging trips) only some 25% of foraging flights, on average, are preceded by dance observation by a forager (Biesmeijer and Seeley, 2005) and in less than half of these cases was a bee recruited to a new food source.

This study also considered whether the recruitment propensity was dependent on the level of foraging experience of the bee and found that even in the case of novice foragers, only about half made use of information acquired from a waggle dance rather than searching independently. Hence, on most foraging trips, bees rely on personal, previously-acquired, knowledge, with even inexperienced foragers relying on trial and error learning (Gruter et al., 2013; Wray et al., 2012).

Although experienced foraging bees are infrequently recruited to a new source of food, they can make use dance

information in a number of ways. The information can be employed as a 'confirmation signal' that the resource they are currently harvesting remains profitable. In this case, the following bee does not need to collect detailed locational information from the dance and typically she only watches the dance briefly before departing on a foraging trip.

The propensity of a bee to use socially-acquired information varies depending on context. If the current food location that a bee is harvesting becomes unprofitable, the forager will 'retire' from it and subsequently look for a new resource either by trial and error search, or by following a dance and being recruited to a new resource location. Hence, experienced bees can employ a flexible strategy – 'copy if dissatisfied' – which combines both personal and social learning, rather than blindly following recruitment dances regardless of feedback to their current behaviour. This strategy is relatively simple to implement as it does not require complex cognition such as a precise comparison of the relative costs and benefits of several alternatives.

Some forager bees also maintain a *memory* of old food sources which they have previously harvested but from which they retired when the source became unprofitable. These 'inspector bees' continue to make occasional trips to the old location to check on its quality and will resume foraging at that location if it again becomes profitable (Biesmeijer and Seeley, 2005). Route memory information to previously harvested resources can be quite persistent and allow a foraging bee to return to food locations even after a gap of some weeks (Gruter and Ratnieks, 2011). Hence, these inspector bees act as short term memory for the bee colony and facilitate the quick reactivation of previously abandoned food sources.

#### 2.4 Summary

From the discussion above, it is evident that the use of social and private information is nuanced in real honey bee foraging behaviours. The majority of foraging bees at any point in time are using private rather than social information (Gruter et al., 2013; Wray et al., 2012), indicating that while social information is important, it does not have the dominant role which it is assigned in most honey bee algorithms. This suggests a number of interesting research questions, including:

- What role does individual perception play in the foraging process?
- What is the impact of noisy recruitment?
- What impact does reliance on private information have on the foraging process?

Addressing these questions could provide some interesting insights into the real-world foraging activities of honey bees by showing which mechanisms are most important, and could also assist in the design of efficient optimisation algorithms. These issues can be considered by designing a series of algorithms which embed (in turn) a 'perception' mechanism, a 'noisy recruitment' mechanism, and finally, a 'private information' mechanism.

#### 3 Model development

In this section, we describe how each of the three mechanisms are implemented in our study. These mechanisms are initially implemented individually, and then tested in various combinations on the benchmark optimisation problems.

#### 3.1 Perception

In implementing this mechanism, we exclude all social information and memory. Hence, each foraging flight starts with no a priori information as to the location of resources. The perception mechanism could be implemented in many ways, depending on the behavioural assumptions made. Real-world foraging bees engage in serial search and stop as soon as a resource of minimal threshold quality is found. In this study, we assume that individual bees fly to a random point in the search space and forage there. This produces a random search process and is used as a benchmark for the next two mechanisms. Whilst this may appear to be a very simple mechanism, it has been noted that where the range of sensory perception in a forager is relatively small compared to the size of the search space, perception-driven search produces little more than a random search process (Viswanathan et al., 2011).

#### 3.2 Noisy recruitment

Under noisy recruitment, in each iteration of the algorithm, the population is ranked in order of the quality of the locations currently being exploited by each bee. The best 20% of the population recruit follower bees. Each follower is then assigned a foraging location which is displaced to a random point within a hypersphere around the foraging location of their recruiter. The radius of this hypersphere (r) is given by

$$r = \frac{R}{3.6\sqrt[D]{N}} \tag{1}$$

where D is the dimensionality of the benchmark problem, R is the radius of the search space, and N is the population size. The constant coefficient is an adjustable parameter. Hence, r is scaled appropriately as the benchmark problem is altered.

#### 3.3 Private information

In the 'private information' mechanism, each bee maintains a personal memory of the best location she has found to date, and subsequently forages at a randomly displaced location within a hypersphere surrounding the foraging location of their previous best location (as above). If a better location is subsequently found, their personal memory is updated to the new location.

#### 3.4 Combinations of mechanisms examined

A total of seven combinations of these mechanisms of honey bee foraging optimisation algorithm (HFOA) are examined as described in Table 1. The canonical versions of the three mechanisms above are denoted as HFOA1 (perception only), HFOA2 (noisy recruitment only) and HFOA3 (private information only), respectively. We note that noisy recruitment and private information are strongly exploitative in nature, as they focus search in the region of previously found resources. In contrast, perception is exploratory in nature, as it is not impacted by the results of any previous searches.

In HFOA12, we combine both individual perception and noisy recruitment (perception and social information). In this variant, 80% of bees are assumed to rely on individual perception, with the remaining 20% of the population being recruited as followers of the best-performing bees. The recruited bees in each iteration of the algorithm are the lowest-ranking 20% of the population in terms of the fitness (quality) of their current foraging location and they are subject to the noisy recruitment mechanism.

HFOA13 combines perception and private information (perception and personal memory). In this combination, 20% of the bees use perception, and 80% continue to forage in the vicinity of the best location they have found to date. The population is ranked in order of fitness at the end of each iteration, and the better-performing bees rely on their private information in the next iteration

HFOA23 combines noisy-recruitment and private information (social information and personal memory). In this combination, 80% rely on private information and 20% are recruited as followers in each iteration. As above, the poorest-performing bees are recruited.

HFOA123 combines all three mechanisms (perception, social information and personal memory), with 16% of bees relying on perception, 20% on recruitment, and 64% on private information.

The proportion of bees allocated to each mechanism in HFOA12–HFOA123, are selected subjectively and alternative weightings could be applied. However, the choice of allocation in HFOA12, HFOA23 and HFOA123 is guided by empirical studies which show that honey bees observe dances only about 25% of the time before undertaking a foraging flight. Hence, the majority of foraging flights, even when social information is available, rely on private information and/or trial and error learning.

	HFOA1	HFOA2	HFOA3	HFOA12	HFOA13	HFOA23	HFOA123
Perception	$\checkmark$	×	×	$\checkmark$	$\checkmark$	×	$\checkmark$
Noisy recruitment	×	$\checkmark$	×	$\checkmark$	×	$\checkmark$	$\checkmark$
Private information	×	×	$\checkmark$	×	$\checkmark$	$\checkmark$	$\checkmark$
Percentage of bees using perception	100%	0	0	80%	20%	0	80%*20%
Percentage of bees recruited	0	100%	0	20%	0	20%	20%
Percentage of bees using private information	0	0	100%	0	80%	80%	80%*80%

#### **Table 1**HFOA algorithm variants

#### Algorithm 1 Pseudocode for HFOA123 algorithm

Randomly locate N foraging bees in the search space;

Evaluate the fitness of each of these locations;

Store location of best solution;

#### repeat

Calculate the fitness of the current location of each bee, and rank the bees according to their fitnesses;

Select  $P_{recruited}$  percentage of the bees with the lowest fitnesses to forage around the  $N * P_{recruited}$  best food locations; each recruiting bee is assigned one follower; **for** *all recruited bees in turn* **do** 

Randomly choose a location around the assigned food site within the hypersphere with radius r, and the bee will forage there;

#### end

Select  $P_{nonpcpt}$  percentage of the unrecruited bees with better fitnesses to forage around their personal best food location (memory);

for all non-perception bees in turn do

Randomly choose a location around its personal best location within the hypersphere with radius *r*,

and the bee will forage there;

#### end

Select  $P_{pcpt}$  percentage of the unrecruited bees to randomly percept in the search space;

for all perception bees in turn do

Randomly choose a location in the search space,

and the bee will forage there;

#### end

Update the personal best location;

Update location of the best solution if necessary;

until terminating condition;

Return best solution found;

In circumstances where a bee is already harvesting a good food resource, they tend to persist with that resource until its quality falls below a threshold level. Hence, in HFOA13, we allocate the majority of the bees to use of personal memory.

The pseudocode of the algorithm which includes all three mechanisms (HFOA123) is outlined in Algorithm 1.

#### 4 Results and discussion

In this section, we describe the experiments undertaken and present the results from these experiments. Twelve benchmark problems (Table 2), at three levels of dimensionality (20, 40 and 60), giving a total of 36 experiments, were used to assess the developed algorithms. In foraging terms, the benchmark problems can be considered as representing resource environments with differing degrees of resource 'patchiness'. The aim in all the experiments is to find the vector of values which minimises the value of the test functions. Hence, we can define the fitness of a solution vector as the value of the test function at that location, with lower values (in this case, as we are minimising) indicating a better quality (or 'fitter') solution.

The first six benchmarks are standard optimisation problems. Two of these functions namely, Sphere and Rosenbrock, represent unimodal problems; and the other four, Ackley, Griewank, Rastrigin and Schwefel, are more complex functions which contain multiple local optima. The last six benchmarks are variants on the above functions in that the global optima are shifted or rotated (shift is given by the parameter o, and the parameter M represents an orthogonal matrix which is used to rotate the function). The net effect of applying these is to move the global optimum away from the origin in each case, due to the known issues with using standard, benchmark functions (Liang et al., 2005), such as:

- many popular benchmark functions are symmetric, and hence have the same optimal parameter values for all dimensions (for example, a vector of zeros)
- the global optimum may lie at the centre of the search space (this can produce problems if search agents are initialised randomly along the range of each dimension).

#### Table 2Optimisation problems

Name	Function	Search space	Optima
Sphere (F1)	$F_1(\mathbf{x}) = \sum_{i=1}^n (x_i - 2)^2$	$[-3.12\ 7.12]^D$	0
Rosenbrock (F2)	$F_2(\mathbf{x}) = \sum_{i=1}^{n-1} \left[ 100 \left( x_{i+1} - x_i^2 \right)^2 + \left( 1 - x_i \right)^2 \right]$	$[-30 \ 30]^{D}$	0
Ackley (F3)	$F_{3}(\mathbf{x}) - 20 \exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D}x_{i}^{2}}\right) - \exp\left(\frac{1}{D}\sqrt{\sum_{i=1}^{D}\cos(2\pi x_{i})}\right) + 20 + e$	$[-32.768 \ 32.768]^D$	0
Griewank (F4)	$F_4(\mathbf{x}) = 1 + \sum_{i=1}^{n} \frac{x_i^2}{4,000} - \prod_{i=1}^{n} \cos\left(\frac{x_i}{\sqrt{i}}\right)$	$[-600 \ 600]^D$	0
Rastrigin (F5)	$F_{5}(\mathbf{x}) = 10n + \sum_{i=1}^{n} \left[ x_{i}^{2} - 10\cos(2\pi x_{i}) \right]$	$[-5.12 \ 5.12]^D$	0
Schwefel (F6)	$F_6(\mathbf{x}) = 418.9829 \times D - \sum_{i=1}^{D} x_i \sin\left( x_i ^{\frac{1}{2}}\right)$	$[-500 \ 500]^D$	0
Shifted Sphere (F7)	$F_7(\mathbf{x}) = \sum_{i=1}^{D} z_i^2 - 450,  \mathbf{z} = \mathbf{x} - 0$	$[-100 \ 100]^D$	-450
Shifted Rosenbrock (F8)	$F_8(\mathbf{x}) = \sum_{i=1}^{D-1} 100 (z_i^2 - z_{i+1})^2 + (x_i - 2)^2 + 390,  \mathbf{z} = \mathbf{x} - 0 + 1$	$[-100 \ 100]^D$	390
Shifted Rotated Ackley (F9)	$F_{9}(\mathbf{x}) = -20 \exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D}z_{i}^{2}}\right) - \exp\left(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi z_{i})\right) +20 + e - 140,  \mathbf{z} = (\mathbf{x} - \mathbf{o}) * M$	$[-32\ 32]^D$	-140
Shifted Rotated Griewank (F10)	$F_{10}(\mathbf{x}) = \sum_{i=1}^{D} \frac{z_i^2}{4,000} - \prod_{i=1}^{D} \cos\left(\frac{z_i}{\sqrt{i}}\right) + 1 - 180,  \mathbf{z} = (\mathbf{x} - 0) * M$	$[-600 \ 600]^D$	-180
Shifted Rotated Rastrigin (F11)	$F_{11}(\mathbf{x}) = \sum_{i=1}^{D} (z_i^2 - \cos(2\pi x_i) + 10) - 330,  \mathbf{z} = (\mathbf{x} - 0) * M$	$[-5 5]^{D}$	-310
Shifted Schwefel (F12)	$F_{12}(\mathbf{x}) = \sum_{i=1}^{D} \left( \sum_{j=1}^{i} z_i \right)^2 - 450,  \mathbf{z} = \mathbf{x} - 0$	$[-100 \ 100]^D$	-450

These features can sometimes be exploited by algorithms to produce an upward bias in reported performance. Hence, considering the convention sphere function,

$$f(x) = \sum_{i=1}^{D} x_i^2,$$

the shifted sphere function is given by:

$$f(x) = \sum_{i=1}^{D} (x_i - o_i)^2,$$

and the shifted rotated sphere function is given by:

$$f(x) = \sum_{i=1}^{D} [(x_i - o_i) * M]^2.$$

Although the focus of this study is not on designing a 'better' optimisation algorithm (rather we wish to better understand the relative contribution of the various search mechanisms), we provide results from the well-known particle swarm optimisation (PSO) algorithm for the same set of problems, in order to provide an illustrative benchmark for the performance of the various algorithms developed.

#### 4.1 Experimental settings

Table 3 describes the parameter settings adopted. In each experiment, N bees in the case of HFOA1-123, or N particles in the case of PSO, are used. We considered two values of N, namely 25 and 50 (giving 72 (2\*36) experiments in total). All reported results are averaged over 30 runs for each problem and algorithm, and we test the statistical significance of all differences in the means using a *t*-test. In all experiments, an equivalent number of function evaluations are undertaken in order to ensure a fair comparison between the different algorithms. The experiments were undertaken on an Intel Core i7 (2.93 GHz) system with 12 GB RAM.

Table 3Parameter settings

Parameters	Values
Trials	30
Size of population	N = 25, 50
Dimension of problem	D = 20, 40, 60

























		20D	40D	60D	20D	40D	60D
PSO	Best	7.9800	58.8482	173.0472	463,612	14,746,605	125,676,300
	Mean	26.0157	128.9998	295.7869	3,900,768	130,401,236	346,439,302
	Std.	15.4472	43.6149	58.9272	13,721,198	73,255,362	118,716,027
HFOA123	Best	0.0157	0.4346	3.8163	96.7029	3,798.2852	258,332
	Mean	0.0278	0.6570	5.9561	572.9255	16,232.0124	786,770
	Std.	0.0079	0.1347	1.2696	845.3591	9,595.2718	378,449
	$H_{pso}^{123}$	0.00	0.00	0.00	0.12	0.00	0.00
	$H_{12}^{123}$	0.00	0.00	0.00	0.01	0.00	0.00
	$H_{23}^{123}$	0.45	0.79	0.76	0.82	0.63	0.41
	$H_{13}^{123}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA12	Best	0.0718	2.1002	19.7364	406.5317	105,325	5,576,346
	Mean	0.1211	3.2682	32.6738	1,010.3170	308,336	10,431,579
	Std.	0.0344	0.7410	6.1932	452.4805	228,208	3,527,660
	$H_{pso}^{12}$	0.00	0.00	0.00	0.12	0.00	0.00
	$H_{1}^{12}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{2}^{12}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA23	Best	0.0122	0.3965	4.1489	110.6861	7,429.0242	337,300
	Mean	0.0293	0.6676	6.0590	622.5741	17,319.0431	872,890
	Std.	0.0077	0.1724	1.3907	868.2932	7,744.1580	425,834
	$H_{pso}^{23}$	0.00	0.00	0.00	0.12	0.00	0.00
	$H_{2}^{23}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{3}^{23}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA13	Best	0.2601	39.6355	93.8218	3,107.0212	10,269,337	104,888,178
	Mean	1.3433	51.0372	142.3633	85,857.3561	28,541,725	138,103,997
	Std.	0.8678	5.5607	16.9299	72,350.6437	8,048,985	15,483,147
	$H_{pso}^{13}$	0.00	0.00	0.00	0.13	0.00	0.00
	$H_{1}^{13}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{3}^{13}$	0.08	0.00	0.00	0.00	0.00	0.00
HFOA1	Best	41.4857	150.1741	269.0678	6,815,019	133,041,077	255,582,090
	Mean	56.5087	173.2407	302.7207	34,344,670	186,184,480	392,637,500
	Std.	5.7147	10.7401	13.7622	10,519,554	26,298,696	41,938,730
	$H^1_{pso}$	0.00	0.00	0.53	0.00	0.00	0.04
	$H_2^1$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_3^1$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA2	Best	0.6258	46.4622	132.1044	11,570.8979	23,183,466	108,855,594
	Mean	2.3129	60.5898	168.4958	275,705	41,089,201	162,314,515
	Std.	1.1196	6.8598	14.5859	322,108	11,838,721	25,015,337
	$H_{pso}^2$	0.00	0.00	0.00	0.15	0.00	0.00
	$H_{3}^{2}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA3	Best	0.2872	40.0317	132.3848	8,308.3104	16,244,203	81,703,246
	Mean	1.7525	60.4401	166.6893	287,917	41,968,517	160,474,859
	Std.	0.9544	8.5872	16.7326	323,575	11,422,752	36,680,524
	$H^3_{pso}$	0.00	0.00	0.00	0.15	0.00	0.00

**Table 4**Results for functions F1 (left three columns) and F2 (right three columns) (N = 50)

Table 5	Results for functions	F3 (left three colum	ns) and F4 (right th	ree columns) ( $N = 50$ )
				/ / /

		20D	40D	60D	20D	40D	60D
PSO	Best	13.1583	18.1097	20.9116	27.9245	178.9409	777.9257
	Mean	17.7009	20.6592	21.1177	82.5563	460.5050	1,081.3291
	Std.	2.7661	0.7427	0.0904	47.7512	145.1415	151.3691
HFOA123	Best	1.4202	4.1139	8.7059	1.0572	2.6195	15.4403
	Mean	2.1231	5.7326	12.7343	1.1012	3.3808	20.6594
	Std.	0.3283	1.1724	2.6508	0.0242	0.4240	3.3894
	$H_{pso}^{123}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{12}^{123}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{23}^{123}$	0.84	0.58	0.36	0.44	0.92	0.20
	$H_{13}^{123}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA12	Best	3.0278	6.5841	14.1055	1.1813	7.4918	71.1612
	Mean	3.5382	10.1791	16.6031	1.3944	12.6905	108.7531
	Std.	0.3075	2.7604	1.3238	0.1095	3.3716	21.0687
	$H_{pso}^{12}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{1}^{12}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{2}^{12}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA23	Best	1.6410	4.3297	9.1314	1.0538	2.5453	14.8038
	Mean	2.1393	5.9000	13.3654	1.0965	3.3936	22.0845
	Std.	0.2931	1.2032	2.6968	0.0242	0.5846	4.9773
	$H_{pso}^{23}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{2}^{23}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{3}^{23}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA13	Best	6.4372	15.8887	17.7776	1.7674	134.6856	378.6470
	Mean	9.2151	17.3825	18.9376	4.7853	179.8782	487.0381
	Std.	1.5283	0.6697	0.3438	1.9208	20.9268	50.1217
	$H_{pso}^{13}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{1}^{13}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{3}^{13}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA1	Best	18.0571	19.7685	20.2208	142.7544	509.2849	934.4449
	Mean	19.2454	20.2075	20.4898	193.5474	582.8584	1,046.3057
	Std.	0.2895	0.1520	0.1074	20.1804	32.9719	52.2187
	$H^1_{pso}$	0.00	0.00	0.00	0.00	0.00	0.23
	$H_2^1$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_3^1$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA2	Best	5.7525	16.8561	18.8042	1.7434	149.2034	443.3706
	Mean	11.2107	18.0306	19.3172	8.0097	206.2267	571.4672
	Std.	2.4401	0.4997	0.2015	3.6460	30.5012	61.9264
	$H_{pso}^2$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{3}^{2}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA3	Best	6.2550	16.3526	18.7761	3.5506	153.3897	412.7703
	Mean	11.4938	18.1017	19.3288	9.5876	210.0254	567.2142
	Std.	2.1262	0.5950	0.2231	3.7378	27.2512	49.4711
	$H^3_{pso}$	0.00	0.00	0.00	0.00	0.00	0.00

		20D	<i>40D</i>	60D	20D	<i>40D</i>	60D
PSO	Best	126.8809	441.8865	768.2017	2,954.5444	8,792.3265	14,362.7610
	Mean	189.3309	519.2829	880.2417	3,685.7902	10,170.2905	16,730.0840
	Std.	24.4953	38.8380	60.9828	318.9380	537.0678	700.2745
HFOA123	Best	2.5756	41.9951	142.9594	807.9302	3,220.2205	6,525.5732
	Mean	4.7697	54.1434	180.1824	2,036.6886	4,810.0728	8,355.6787
	Std.	1.2255	/.1029	16.8274	510.2760	/62.3563	962.4858
	$H_{pso}^{123}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{12}^{123}$	0.00	0.00	0.00	0.06	0.01	0.00
	$H_{23}^{123}$	0.92	0.00	0.52	0.03	0.48	0.49
	$H_{13}^{123}$	0.00	0.00	0.00	0.09	0.00	0.00
HFOA12	Best	8.0406	93.2518	227.4998	1,327.7092	4,031.2199	7,653.6044
	Mean	14.5655	119.5681	286.0803	2,257.9313	5,312.4719	9,456.8859
	Std.	2.7759	15.9073	27.2696	408.2927	806.5628	867.6762
	$H_{pso}^{12}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{1}^{12}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{2}^{12}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA23	Best	3.1634	44.7158	149.0648	1,600.3065	3,083.9446	6,038.7390
	Mean	4.7409	60.2021	183.0248	2,312.2361	4,947.7597	8,186.0147
	Std.	1.0125	9.6453	17.4333	457.6749	757.3324	948.8448
	$H_{pso}^{23}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{2}^{23}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{3}^{23}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA13	Best	30.4592	163.7951	364.9235	1,222.1671	4,348.8223	9,610.7241
	Mean	41.5145	204.9313	422.6800	1,859.7164	5,559.1591	10,746.1472
	Std.	5.4971	15.6158	23.9450	252.1411	459.7526	571.7706
	$H_{pso}^{13}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{1}^{13}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{3}^{13}$	0.00	0.05	0.00	0.52	0.07	0.40
HFOA1	Best	156.2712	465.0008	746.1760	4,281.2210	11,181.2195	17,367.7513
	Mean	194.0410	499.8018	804.9176	4,995.0158	12,034.1978	19,407.7954
	Std.	13.4148	14.0845	21.8305	278.8728	349.4736	502.0749
	$H^1_{pso}$	0.35	0.01	0.00	0.00	0.00	0.00
	$H_2^1$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_3^1$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA2	Best	32.3377	182.7121	387.5177	621.1851	4,924.6477	9,407.1645
	Mean	47.3489	218.5591	451.5954	1,837.2916	5,894.6220	10,603.5336
	Std.	7.8569	19.6116	27.3821	360.6223	382.5464	596.3492
	$H^2_{pso}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{3}^{2}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA3	Best	31.0905	178.1448	409.4850	1,147.5308	4,742.5557	10,087.4225
	Mean	50.4512	213.6301	453.8173	1,905.7916	5,753.9104	10,864.4330
	Std.	7.4756	19.1753	22.8871	307.5824	370.2067	511.6868
	$H^3_{pso}$	0.00	0.00	0.00	0.00	0.00	0.00

**Table 6**Results for functions F5 (left three columns) and F6 (right three columns) (N = 50)

**Table 7**Results for functions F7 (left three columns) and F8 (right three columns) (N = 50)

		20D	40D	60D	20D	40D	60D
PSO	Best	2,110.9006	25,462.9793	75,498.4190	159,176,631	5,740,162,399	43,833,683,694
	Mean	10,603.1116	75,473.5576	140,351.5558	2,434,098,394	39,881,615,398	106,624,138,962
	Std.	9,774.7642	20,955.8415	30,541.6187	3,487,358,785	27,868,706,232	44,719,158,381
HFOA123	Best	-445.1319	-276.5322	4,011.1435	1,932.0086	1,204,014	165,567,551
	Mean	-436.9680	33.3043	7,217.1130	8,519.2092	6,719,820	842,984,306
	Std.	3.7472	122.7849	2,083.3755	5,382.8834	4,601,504	433,697,114
	$H_{pso}^{123}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{12}^{123}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{23}^{123}$	0.24	0.46	0.53	0.99	0.12	0.09
	$H_{13}^{123}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA12	Best	-423.3875	889.3842	18,322.1382	30,878.3847	26,838,987	3,033,011,058
	Mean	-389.9918	2,897.7925	29,585.2187	173,401.0776	320,139,830	5,818,667,452
	Std.	15.9675	1,256.7150	4,720.8602	228,045.9205	226,860,586	1,720,970,567
	$H_{pso}^{12}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{1}^{12}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{2}^{12}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA23	Best	-442.6971	-152.7375	3,034.4764	1,887.5437	1,488,889	251,755,224
	Mean	-435.6450	61.0549	7,608.2128	8,511.4050	8,868,983	1,089,929,786
	Std.	4.9036	166.8338	2,770.8777	5,427.6631	5,996,864	679,439,551
	$H_{pso}^{23}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{2}^{23}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{3}^{23}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA13	Best	-187 0656	26 728 3419	90 032 4255	2 322 082	8 579 314 134	49 760 634 537
	Mean	2.129.2258	45.013.8887	109.802.3124	179.920.914	20.900.833.094	72.024.813.741
	Std.	1.413.4333	8.009.2251	8.553.4358	128.310.886	5.521.338.853	12.214.929.903
	$H_{pso}^{13}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{1}^{13}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{3}^{13}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA1	Best	22,890,3503	85,199,4477	135.811.1990	5.081.937.415	50.481.313.319	95.740.720.772
	Mean	29,016.0357	101,828.8488	175,625.9112	9,275,794,221	64,966,684,899	135,899,052,011
	Std.	3,380.1142	7,027.3520	15,429.9383	2,668,554,157	8,775,335,926	16,443,962,042
	$H^1_{pso}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_2^1$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_3^1$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA2	Best	-159.8496	27.171.9193	81.884.9283	20.310.740	11.470.787.135	70.299.585.812
	Mean	4,328.8876	54,871.2141	132,750.9940	787,443,974	33,108,568,428	109,718,175,335
	Std.	2,336.3970	9,927.2089	18,795.5413	607,938,994	10,288,815,732	22,176,940,746
	$H_{pso}^2$	0.00	0.00	0.25	0.01	0.21	0.73
	$H_3^2$	0.70	0.52	0.56	0.32	0.52	0.21
HFOA3	Best	-221.1827	40.246.0386	91.145.7692	27.519.448	9.925.332.831	48,844,359,824
	Mean	4.108.5589	56.517 4293	130.068 4473	646 239 765	31.312.268.917	102.844 051 680
		.,100.000	10.041.0(70	16.026.7052	475 000 977	11 272 200,007	20.026.964.605
	Std.	2.210.3463	10.241.0672	16,836.7953	4/3.009 8/7	11.5/3.800.077	20.020.804 095

		20D	40D	60D	20D	40D	60D
PSO	Best	-119.3423	-119.0032	-118.7846	72.0126	1,812.1492	5,368.5261
	Mean	-119.0968	-118.8190	-118.6971	1,317.2877	4,504.7978	8,391.8754
1150 4 102	Std.	-119.0968	0.0624	0.0355	741.6712	940.4476	1,635.6794
HFOA123	Best	-119.5421	-118.96/9	-118.8364	-1/8.8512	-1/4.3960	-163.5678
	Std	-119.1260	-118.8339	-118.7255	-177.0637	-115.9309	78.0223
	$H_{nso}^{123}$	0.25	0.30	0.02	0.00	0.00	0.00
	$H_{123}^{123}$	0.64	0.40	0.91	0.00	0.00	0.00
	$H_{22}^{123}$	0.13	0.80	0.99	0.00	0.02	0.31
	$H_{12}^{123}$	0.14	0.29	0.12	0.00	0.00	0.00
HEOA12	Best	_110 3338	_118 93/0	_118 8248	_178 7985	-168 9282	_86 7989
III OA12	Mean	-119.1140	-118.9340	-118 7221	-175 2998	-49.0638	676 7891
	Std.	0.0860	0.0440	0.0412	3.7174	116.1276	681.5311
	$H_{pso}^{12}$	0.43	0.73	0.01	0.00	0.00	0.00
	$H_{1}^{12}$	0.44	0.29	0.07	0.00	0.00	0.00
	$H_{2}^{12}$	0.37	0.41	0.26	0.00	0.00	0.00
HFOA23	Best	-119.2645	-118.9629	-118.7771	-178.9125	-176.4055	-158.6304
	Mean	-119.0895	-118.8304	-118.7233	-178.0174	-135.2982	129.7135
	Std.	0.0703	0.0551	0.0279	0.9018	45.4288	305.6116
	$H_{pso}^{23}$	0.71	0.45	0.00	0.00	0.00	0.00
	$H_{2}^{23}$	0.60	0.21	0.13	0.00	0.00	0.00
	$H_{3}^{23}$	0.54	0.99	0.26	0.00	0.00	0.00
HFOA13	Best	-119.2523	-118.9181	-118.7968	-171.9703	-28.2328	1,135.8155
	Mean	-119.0933	-118.8215	-118.7067	-109.0682	829.1227	2,301.2468
	Std.	0.0519	0.0411	0.0307	63.7647	599.9658	971.7985
	$H_{pso}^{13}$	0.84	0.85	0.26	0.00	0.00	0.00
	$H_{1}^{13}$	0.86	0.37	0.68	0.00	0.00	0.00
	$H_{3}^{13}$	0.65	0.42	0.45	0.00	0.00	0.00
HFOA1	Best	-119.3487	-118.9719	-118.8304	506.1442	2,155.2874	4,876.5842
	Mean	-119.0965	-118.8113	-118.7028	918.4354	3,208.1608	5,927.9785
	Std.	0.0895	0.0470	0.0413	154.0620	366.7085	418.0055
	$H^1_{pso}$	0.98	0.59	0.56	0.00	0.00	0.00
	$H_2^1$	0.94	0.75	0.41	0.00	0.00	0.00
	$H_3^1$	0.83	0.11	0.31	0.00	0.00	0.00
HFOA2	Best	-119.1768	-118.9061	-118.7794	-170.1390	136.7807	1,350.7470
	Mean	-119.0978	-118.8149	-118.7110	-81.2645	1,317.4547	3,181.6124
	Std.	0.0507	0.0398	0.0347	93.2936	879.1889	1,293.6837
	$H_{pso}^2$	0.95	0.76	0.13	0.00	0.00	0.00
	$H_{3}^{2}$	0.85	0.16	0.79	0.21	0.94	0.91
HFOA3	Best	-119.3414	-118.9344	-118.7978	-173.7923	220.6589	850.3924
	Mean	-119.1009	-118.8304	-118.7135	-56.8271	1,306.9859	3,156.8147
	Std.	0.0752	0.0448	0.0391	120.2365	860.3211	1,291.9427
	$H^3_{pso}$	0.84	0.41	0.09	0.00	0.00	0.00

**Table 8**Results for functions F9 (left three columns) and F10 (right three columns) (N = 50)

Table 9	Results for functions F11	(left three columns)	) and F12 (right three c	olumns) $(N = 50)$
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		20D	40D	60D	20D	40D	60D
PSO	Best	-180.5801	120.7483	906.1210	10,847	122,080	411,329,808
	Mean	-122.8302	435.5849	1,211.6267	52,316	331,984	1,301,840
	Std.	43.9232	144.9717	167.5247	30,837	163,960	1,054,256
HFOA123	Best	-171.8053	226.6004	794.8756	784	26,422	68,197
	Mean	-99.8343	472.7583	1,085.2722	4,429	41,655	103,060
	Std.	41.9081	97.0205	110.6729	2,142	11,207	17,196
	$H_{pso}^{123}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{12}^{123}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{23}^{123}$	0.00	0.00	0.00	0.33	0.29	0.81
	$H_{13}^{123}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA12	Best	-164.0436	265.9740	779.2154	1,775	31,149	88,129
	Mean	-93.1168	423.4131	975.8545	8,107	57,931	135,173
	Std.	31.3641	90.1987	110.5335	3,307	15,185	27,044
	$H_{pso}^{12}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{1}^{12}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{2}^{12}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA23	Best	-194.9120	394.1739	888.7401	1,001	27,006	60,073
	Mean	-30.1095	541.9228	1,138.9803	5,093	44,543	101,895
	Std.	63.1312	78.3011	136.2489	3,083	10,131	20,128
	$H_{pso}^{23}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{2}^{23}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{3}^{23}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA13	Best	-93.1971	294.8667	973.1546	7,441	51,763	125,451
	Mean	-54.9156	432.7575	1,169.1984	14,571	72,304	175,202
	Std.	26.1165	58.5234	87.9053	3,215	8,786	20,512
	$H_{pso}^{13}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{1}^{13}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{3}^{13}$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA1	Best	-42.4794	515.3253	1,149.2543	27,317	92,056	233,013
	Mean	32.5078	644.1997	1,283.2124	35,647	132,720	309,197
	Std.	29.2464	57.1423	65.0759	4,794	17,135	37,705
	$H^1_{pso}$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_2^1$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_3^1$	0.00	0.00	0.00	0.00	0.00	0.00
HFOA2	Best	-113.9786	475.6374	1060	10,921	46,320	167,197
	Mean	10.4027	709.0625	1,382.7171	19,117	85,973	218,629
	Std.	54.0935	104.4768	120.6309	4,230	13,987	25,165
	$H_{pso}^2$	0.00	0.00	0.00	0.00	0.00	0.00
	$H_{3}^{2}$	0.00	0.00	0.00	0.78	0.63	0.34
HFOA3	Best	-106.2963	453.1859	951.3757	11,112	64,806	164,006
	Mean	19.4954	620.9922	1,307.3017	18,839	87,465	212,002
	Std.	50.9533	78.1252	156.7817	3,673	10,103	28,388
	$H^3_{pso}$	0.00	0.00	0.00	0.00	0.00	0.00

#### 4.2 Presentation of results

Figures 1 to 6 and Tables 4 to 9 present the results from our experiments. As the results for the case where N = 25 were qualitatively similar to those of N = 50, we only provide the results for the latter in order to conserve space. The tables show for each algorithm – problem combination, the end of run evaluation of each benchmark function at the best location (solution vector) found across all 30 runs ('best'), and the evaluation of each benchmark function averaged over the best location (solution vector) found on each of the 30 individual runs ('mean'), and the associated standard deviation of each benchmark function at 'mean' and indicate how this value changes (improves) as the number of iterations increases.

Tables 4 to 9 also present the results from our statistical testing of a variety of hypotheses. In all cases, the null hypothesis is that there is no difference in performance between the algorithms being compared. Hence, low p values indicate that the null hypothesis is rejected (a 95% level is applied).

In order to facilitate interpretation of the statistical tests, we now outline the notation used. One set of hypotheses examine the comparative performance of the HFOA variants with the canonical PSO algorithm. The notation adopted here is:

•  $H_{pso}^i$ : where the mean result from HFOAi (*i*: 1, 2, 3, 12, 23, 13, 123) is compared with the mean result from PSO algorithm (null hypothesis of no difference).

In all the other hypotheses, we compare two variants of the HFOA algorithm. The notation in all of these hypotheses is illustrated using the following two examples:

- $H_{23}^{123}$ : null hypothesis of no difference between HFOA123 algorithm and HFOA23 algorithm
- $H_2^{12}$ : null hypothesis of no difference between HFOA12 algorithm and HFOA2 algorithm.

#### 4.3 Discussion of results

Initially, we overview Figures 1 to 6 to get an idea of the general trends in the results. Taking a high-level perspective, we note that the performance of the HFOA variants generally improve as they embed more features, with HFOA123 generally performing best, closely followed by HFOA23. As would be expected, HFOA1 is typically the weakest performer. As the dimensionality of the search space increases from 20 to 60, the general ordering of the performance of the HFOA variants remains similar, although as would be expected, the absolute performance of each algorithm variant tends to decrease as the problem becomes more difficult. Looking at the results for PSO, the performance of the algorithm tends to decrease relative to that of the better performing HFOA variants as the dimensionality of the problem increases. Difficulties with search space scalability in the canonical PSO algorithm are

well-known, and hence this trend in the results is not surprising. Next, we proceed to look at the results in Tables 4 to 9 in order to obtain finer detail.

#### 4.3.1 Perception

Looking at Tables 4 to 9, it is noted that perception alone (HFOA1) does not perform particularly well, in comparison with the other HFOA variants. HFOA2 outperforms HFOA1 in terms of mean fitness (i.e., mean benchmark function evaluation) on 34 of the 36 experiments and the difference in the means was statistically significant in all of these cases. HFOA3 outperforms HFOA1 in terms of mean fitness on 35 of the 36 experiments, and in 32 cases the difference is statistically significant.

The relatively modest performance of HFOA1 vs. that of HFOA2 and HFOA3 is not particularly surprising, as HFOA1 amounts to random search, with a memory being kept of the best location found by any bee in the population. It is also notable, that the standard deviation of the results obtained by HFOA1 tends to be high, which is again unsurprising given the expected variability of results from a random search mechanism.

#### 4.3.2 Social vs. private information

Comparing the performance of HFOA2 and HFOA3, the results produced by each algorithm are qualitatively similar over most problems in terms of mean performance, and each algorithm produces similar-sized standard deviations. In numeric terms, HFOA3 outperforms HFOA2 in terms of mean result in 17 cases (in eight cases the difference is statistically significant), and HFOA2 outperforms HFOA3 on the remaining 19 cases (in 13 cases the difference is statistically significant). This suggests that use of social information very weakly outperforms the use of private information.

## 4.3.3 Combining perception with social and private information

Considering the combinations of mechanisms, when perception is combined with either social information (HFOA12) or private information (HFOA13), the combination almost always outperformed perception (HFOA1) alone. In the case of social information (HFOA12), better mean results are obtained on all 36 experiments, and in 33 of the 36 experiments the differences in performance were statistically significant. In the case of private information (HFOA13), better results were obtained on 35 of the 36 experiments (33 of these produced statistically significant results).

When perception is combined with social information (HFOA12), the combination outperforms social information (HFOA2) alone in 35 out of 36 experiments (statistically significant in 32 of these cases), and the difference is statistically significant in most cases. When perception is combined with private information (HFOA13), the combination outperforms private information (HFOA3)

alone in 33 out of 36 experiments (statistically significant in 29 of these cases).

The evidence from the above results, suggests that combinations of perception and either social or private information lead to better search outcomes than reliance on only social or private information or perception alone.

#### 4.3.4 Combining social and private information

HFOA23 combines both social information and private information. The combination outperforms social information (HFOA2) alone in 34 out of 36 experiments (statistically significant in 32 of these cases). The combination outperforms private information (HFOA3) alone in 34 out of 36 experiments (one experiment produces a tied result) (statistically significant in 33 of these cases).

These results suggest that a combination of both private and social information produces better search outcomes than reliance on either mechanism in isolation.

#### 4.3.5 Combining all three mechanisms

Next, we consider the performance of the combination of all three mechanisms (HFOA123) when compared with that of HFOA12, HFOA13 and HFOA23. In the case of HFOA12, the results from HFOA123 are better in 34 of 36 experiments (statistically significant in 30 of these cases). In the case of HFOA13, the results from HFOA123 are better in 34 of 36 experiments (statistically significant in 31 of these cases). Comparing HFOA123 against HFOA23, HFOA123 outperforms HFOA23 in 27 experiments (in only four cases is the difference statistically significant), HFOA performs best in eight experiments (in only one case is the difference statistically significant) and one experiment produces a tied result.

These results suggest that the addition of perception to a combination of social and private information produces some improvement in performance, but in most cases the improvement is too small to be statistically significant.

#### 4.3.6 Comparison with PSO

Although the purpose of this paper is primarily to gain insight into the relative importance of three mechanisms in honey bee foraging, we provide some comparative PSO results for the benchmark problems for illustrative purposes. Generally, the combined HFOA variants perform better than PSO. Considering HFOA123, this algorithm variant outperforms PSO on 34 of the 36 experiments (statistically significant in 31 of these cases). Qualitatively similar results are obtained when PSO is compared with HFOA23 and HFOA12. We emphasise that we have not attempted to optimally tune the parameters for either the developed algorithms nor for PSO and hence these results only indicate that the performance of the algorithms studied appears reasonably competitive from an optimisation perspective.

#### 5 Conclusions

In this study, we draw inspiration from the honey bee literature in biology and identify three features of the foraging process of honey bees, namely 'perception', 'noisy recruitment', and 'private information'. We then examine the relative importance of each of these mechanisms for the foraging process using a series of benchmark problems. Our results indicate that both social information and private information are important in guiding the search process, in contrast to the particular importance accorded to social information in existing honey bee algorithms. We also find that a combination of these mechanisms produces better results than a search process that relies completely on either social or private information alone. This underscores the importance of memory in the honey bee foraging process. In contrast, memoryless individual perception is found to be much less effective. Our results suggest that work should be undertaken to examine the utility of honey bee algorithms which explicitly incorporate private memory, in addition to recruitment mechanisms.

Apart from the contribution of this study to our understanding of the importance of specific mechanisms for the design of honey bee optimisation algorithms, the results of our study also provide simulation support for observational studies reported in the honey bee literature which indicate that bees do not place complete reliance on socially transmitted information when selecting a foraging location. To the best of our knowledge, this is the first study which demonstrates this.

A number of avenues for future work are indicated. In this study, we have implemented a very basic version of perception. A follow on study could operationalise this mechanism in more complex ways, for example, allowing a serial step by step search process. As with all simulation studies, alternative parameter values could be chosen and alternative benchmark functions employed. Future work could undertake additional study of this matter. It would also be interesting to investigate the implications of noisy recruitment further. This could be undertaken by implementing a variant algorithm which has noiseless recruitment and determining the impact of this on search effectiveness and efficiency.

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